Two-dimensional morphological simulation in transcritical flow

J.A. Vasquez & R.G. Millar  
Department of Civil Engineering, University of British Columbia, Canada

P.M. Steffler  
Department of Civil and Environmental Engineering, University of Alberta, Canada

ABSTRACT: A two-dimensional Finite Element hydrodynamic model with capabilities for transcritical flow has been coupled with a sediment transport model to simulate bed elevation changes in alluvial beds. The model has been tested for a knickpoint migration experiment that involved a short and very steep reach of 10% bed slope. The initial Froude was Fr = 3.5 downstream from the knickpoint with a hydraulic jump formed close to the toe of the oversteepened reach. The results computed by the model agreed with the experimental data. The new upstream location of the knickpoint was correctly predicted and the computed bed profile seemed to follow the average trend of the measured profile. The model seems as a promising morphodynamic tool for alluvial streams subject to transcritical flow.

1 INTRODUCTION

Although several river morphology models are currently available, most of them are intended for applications in lowland alluvial rivers with small gradients (Papanicolaou, in press). Such models usually assume subcritical flow conditions and therefore cannot be applied in cases where supercritical flow or hydraulic jumps are present.

A transcritical flow condition, where flow regime changes between subcritical and supercritical, usually involves the presence of a sharp shock wave or hydraulic jump, as sketched in Figure 1.

Some practical examples of transcritical flow over alluvial beds are: sediment deposition upstream of dams (Brusnelli et al. 2001, Bellal et al. 2003); dam break (Fraccarollo & Capart 2002, Spinewine & Zech 2002); dam removal (Cantelli et al. 2004, Wong et al. 2004, Cui & Wilcox, in press, Cui et al. a, b, in press); dyke breach (Spinewine et al. 2004); and knickpoint migration (Brush & Wolman 1960, Bhallamudi & Chaudhry 1991).

Figure 1. Sketch of transcritical flow after dam removal (Cui & Wilcock, in press).

Knickpoints are points along longitudinal profiles of streams where the slope increases abruptly from mild to steep (Brush & Wolman 1960). Knickpoints may appear after meander cut-off (Brush & Wolman 1960) or dam removal (Fig. 1). Their presence can lead to transcritical flow conditions and intense scour and deposition.

Morphodynamic models with transcritical flow capabilities are uncommon, and normally limited to one-dimensional (1D) flow (e.g. Papanicolaou, in press, Brusnelli et al. 2001, Cui et al. a, b, in press).

The main objective of this paper is to assess the capabilities of a new two-dimensional (2D) depth-averaged river morphology model for simulating scour and deposition in transcritical flow generated by a knickpoint. Comparisons with the experimental data of Brush & Wolman (1960) suggested that model is a promising tool for alluvial streams subject to transcritical flow.

2 THE NUMERICAL MODEL

2.1 River2D hydrodynamic model

River2D is a two-dimensional (2D) depth-averaged hydrodynamic model intended for use on natural streams and rivers and has special features for supercritical/subcritical flow transitions, ice covers, and variable wetted area. For the spatial discretization it uses a flexible Finite Element unstructured mesh composed of triangular elements. River2D is based on the 2D vertically averaged Saint Venant equations.
expressed in conservative form; which form a system of three equations representing the conservation of water mass and the two components of the momentum vector (Steffler and Blackburn, 2002). It has been developed at the University of Alberta and it is freely available at www.River2D.ca.

The Finite Element Method used in River2D’s hydrodynamic model is based on the Streamline Upwind Petrov-Galerkin (SUPG) weighted residual formulation (Hicks & Steffler 1992, Ghanem et al. 1995). In this technique, upstream biased test functions are used to ensure solution stability under the full range of flow conditions, including subcritical, supercritical, and transcritical flow. A fully conservative discretization is implemented which ensures that no fluid mass is lost or gained over the modeled domain. This also allows implementation of boundary conditions as natural flow or forced conditions. The equations solved by the River2D are (Steffler and Blackburn 2002):

The water continuity equation:

\[
\frac{\partial h}{\partial t} + \frac{\partial q_x}{\partial x} + \frac{\partial q_y}{\partial y} = 0 \tag{1}
\]

The vertically averaged momentum equation in the \(x\)-direction:

\[
\frac{\partial q_x}{\partial t} + \frac{\partial (u q_x)}{\partial x} + \frac{\partial (v q_x)}{\partial y} + g \frac{\partial h^2}{\partial x} = \rho g h (S_{ax} - S_{fx}) + \frac{1}{\rho} \left( \frac{\partial (h \tau_{xx})}{\partial x} + \frac{\partial (h \tau_{yx})}{\partial y} \right) \tag{2}
\]

The vertically averaged momentum equation in the \(y\)-direction:

\[
\frac{\partial q_y}{\partial t} + \frac{\partial (u q_y)}{\partial x} + \frac{\partial (v q_y)}{\partial y} + g \frac{\partial h^2}{\partial y} = gh (S_{ay} - S_{fy}) + \frac{1}{\rho} \left( \frac{\partial (h \tau_{yx})}{\partial x} + \frac{\partial (h \tau_{yy})}{\partial y} \right) \tag{3}
\]

where \(h\) = water depth; \((u, v)\) = depth-averaged velocities in the \((x,y)\) directions; \(q_x = uh = \) flow discharge in \(x\)-direction per unit width; \(q_y = vh = \) flow discharge in the \(y\)-direction per unit width; \((S_{ax}, S_{ay}) = \) bed slopes in the \((x,y)\) directions; \(S_{fx}\) and \(S_{fy}\) = corresponding friction slopes; \(\tau_{xx}, \tau_{yx}, \tau_{xy}, \tau_{yy} = \) components of the horizontal turbulent stress tensor; \(\rho = \) water density; and \(g = \) gravitation acceleration.

The basic assumptions in equations (1) through (3) are (Steffler and Blackburn 2002):

- The pressure distribution is hydrostatic, which limits the accuracy in areas of steep slopes and rapid changes in bed slopes.
- The horizontal velocities are constant over the depth. Information on secondary flows and circulations is not available.
- Coriolis and wind forces are assumed negligible.

The friction slope terms depend on the bed shear stresses which are assumed to depend on the magnitude and direction of the vertically averaged velocities. For example, in the \(x\)-direction:

\[
\tau_{bx} = \frac{\tau_{bx}}{\rho g h} = \frac{\sqrt{u^2 + v^2}}{gh C_*^2} \tag{4}
\]

\(\tau_{bx}\) is the bed shear stress in the \(x\) direction and \(C_*\) is the dimensionless Chezy coefficient, which is related to the effective roughness height \(k_*\) through

\[
C_* = 5.75 \log \left( \frac{12 h}{k_*} \right) \quad ; \quad \frac{h}{k_*} \geq \frac{e^2}{12} \tag{5a}
\]

\[
C_* = 2.5 + \frac{30}{e^2} \left( \frac{h}{k_*} \right) \quad ; \quad \frac{h}{k_*} < \frac{e^2}{12} \tag{5b}
\]

\(e = 2.7182\); \(C_*\) is related to Chezy’s \(C\) coefficient through

\[
C_* = \frac{C}{\sqrt{g}} \tag{6}
\]

The vertically-averaged turbulent shear stresses are modeled with a Boussinesq type eddy viscosity. For example:

\[
\tau_{xy} = \nu_t \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) \tag{7}
\]

where \(\nu_t = \) eddy viscosity coefficient assumed by default as

\[
\nu_t = 0.5 \frac{h}{C_*} \left( \frac{\sqrt{u^2 + v^2}}{C_*} \right) \tag{8}
\]

Unlike other popular 2D hydrodynamic models based of Finite Elements (e.g. RMA2, FESWMS), River2D does not require artificial eddy viscosity for convergence. In fact, River2D is rather insensitive to the value of eddy viscosity.

2.2 River2D-Morphology

River2D was extended by including a solver for the bed load sediment continuity (Exner) equation

\[
(1 - \lambda) \frac{\partial q_{b}}{\partial t} + \frac{\partial q_{bx}}{\partial x} + \frac{\partial q_{by}}{\partial y} = 0 \tag{9}
\]
where $z_b = \text{bed elevation}$; $\lambda = \text{porosity of the bed material}$; $t = \text{time}$; and $(q_{sx}, q_{sy}) = \text{components of the volumetric rate of bedload transport per unit width } q_s \text{ (ignoring the effects of secondary flow and bed slope)}$:

$$q_{sx} = \frac{u}{\sqrt{u^2 + v^2}} q_s$$  \hspace{1cm} (10a)

$$q_{sy} = \frac{v}{\sqrt{u^2 + v^2}} q_s$$  \hspace{1cm} (10b)

Equation (9) is discretized using a conventional Galerkin Finite Element Method (GFEM). The time advance is performed using a Runge-Kutta second order scheme. Since GFEM lacks upwinding properties (equivalent to a central difference scheme) it is known to be adequate for diffusive problems, but not for advection-dominated problems (Hirsch 1987, Chung 2002, Donea & Huerta 2003).

This extended version is known as River2D-Morphology and has been successfully applied to simulate aggradation, degradation and bend scour in alluvial channels (Vasquez et al. 2005a, b). However, the model should not be applied to problems where advection is important, such as migrating bedforms or sediment waves.

3 EXPERIMENTAL TEST CASE

3.1 Experimental data

Brush & Wolman (1960) carried out a laboratory study of the behavior of knickpoints in noncohesive sediment. The experiments were conducted at the University of Maryland in a flume 15.85 m (52 ft) long and 1.22 m (4 ft) wide. Before starting each experimental run a trapezoidal channel 0.21 m (0.7 ft) wide and 0.03 m (0.1 ft) deep with rounded corners was molded in noncohesive sand. A short over-steepened reach with a length of 0.30 m (1 ft) and a fall of 0.03 m (0.1 ft), was located 10.8 m from the flume entrance. This oversteepened reach immediately downstream of the knickpoint had a 10% slope which was significantly steeper than the adjacent reaches. A total of 5 runs were performed with different slopes in the upstream and downstream reaches (between 0.125 and 0.88%). In every run, the slope of both water surface and bed below the knickpoint decreased with time, causing the position of the knickpoint to migrate upstream. As the knickpoint moved upstream the channel directly above it narrowed. At the lower end of the steep reach, sediment eroded from above deposited as a dune, which moved downstream causing the channel to locally widen.

Run 1, which has more detailed information, was selected for the numerical simulation. For Run 1 the longitudinal bed slope changed at a knickpoint from 0.125% to 10% and back to 0.125% again (Fig. 3). The channel was molded in sand 0.67 mm in diameter with initial water depth $h_0 = 0.0137 \text{ m}$ and constant flow rate $Q = 0.59 \text{ L/s}$. During Run 1 the width of the channel increased downstream of the knickpoint by bank erosion to about 0.25 m; while it got deeper and narrower upstream.

At the beginning of Run 1, no sediment was moving in the reaches above and below the steep reach. The erosion and deposition resulted solely from the presence of the knickpoint. Sediment eroded above the knickpoint was deposited below. The head and toe of the steep reach moved upstream and downstream respectively. Sediment transport rate was not measured.

3.2 Numerical model

In the numerical model the channel was assumed as rectangular with a fixed width of 0.21 m. An irregular mesh was used; the average size of the elements was about 0.20 m away from the steep reach and about 0.04 m around the knickpoint (Fig. 2). The mesh was refined close to the knickpoint to capture the strong flow gradients and the hydraulic jump. The mesh had 869 triangular elements and 563 nodes. The roughness height was set to $k_s = 0.3 \text{ mm}$ and porosity to $\lambda = 0.4$.

The boundary conditions for the hydrodynamic model were constant discharge in the upstream inflow section, and constant water surface elevation in the downstream outflow section.

4 RESULTS

Figure 3 shows the initial water surface profile computed before the beginning of the morphological simulation. Away from the steep reach, the flow remained subcritical with a Froude around 0.5; which corresponds to a velocity of about 0.2 m/s and a depth of 0.014 m, in agreement with observed values (Table 3 in Brush & Wolman 1960). As the flow approaches the knickpoint, it accelerates and becomes supercritical. The maximum velocity of about 0.7 m/s is reached
over the steep reach, the Froude number being as high as 3.5 with a water depth of only 0.004 m.

Downstream from the toe of the steep reach, the flow returns to subcritical through a hydraulic jump. The sharp front of the jump was captured by the model, as shown in Figure 3, without any noticeable spurious oscillation.

Since most sediment transport equations are derived for subcritical conditions, there is considerable uncertainty in the sediment transport equation for this experiment. Some preliminary runs were made testing 3 different sediment transport equations: Engelund-Hansen, Van Rijn and the empirical equation used by Bhallamudi & Chaudhry (1991) for this particular experiment:

$$q_s = (S_f)^{1.7} \left( \frac{\nu^2}{\nu^2 + v^2} \right)^{4.2}$$

(11)

None of the equations provided a perfect fit with the measured data; but Engelund and Hansen and Van Rijn equations notoriously underpredicted the upstream migration of the knickpoint (results not shown). Therefore, equation (11) was adopted.

A very small time step was used $\Delta t = 0.01$ s to prevent large bed changes during the initial period of the simulation. The results showed that both the computed bed and water surface profiles were rather smooth, without any noticeable numerical oscillation.

Figure 4 shows the computed bed and water surface profiles at the early stages of the simulation. It can be noticed that the most dramatic changes happen during the first minute. The profiles seem to pivot around the middle point of the steep reach. The intensity of the hydraulic jump decreases rapidly. The toe of the steep reach and the hydraulic jump moved rapidly downstream; while the knickpoint migrated upstream.

A comparison between the computed profiles and the bed profiles measured by Brush & Wolman (1960) at $t = 160$ min is shown in Figure 5. By this time, the intensity of the hydraulic jump has notoriously decreased since the maximum value of the Froude number was only slightly larger than 1.0. The steep bed slope has reduced to only 0.44% from its original value of 10%. The computed bed profile was smoother than the observed profile and tended to follow the average trend of the measurements. The new upstream location of the knickpoint agreed well with the observed value.

Bhallamudi & Chaudhry (1991) also simulated this experiment using a 1D model. However, they did not observe a hydraulic jump in their numerical simulations (Bhallamudi, pers. comm.); likely because the space discretization used $\Delta x = 0.30$ m was too large to capture the sharp front of the jump, and also because their downstream water level was higher ($h_o = 0.03$ m).

5 DISCUSSION

The morphodynamic simulation of the knickpoint migration experiment is a challenging problem because of the steep 10% bed slope that gives rise to a hydraulic jump and transcritical flow conditions.
1D morphological model with transcritical flow capabilities exist (e.g. Busnelli et al. 2001, Fraccarollo & Capart 2002, Papanicolaou et al., in press, Cui et al. a,b, in press); but they are still rare or inexistent in 2D or 3D models (Spinewine & Zech 2002). The shock-capturing techniques incorporated in River2D, added to the flexibility of the Finite Element method to increase mesh resolution around areas with strong gradients (Fig. 2), made possible to simulate the sharp hydraulic jump front (Fig. 3), without any numerical oscillation. This test case would be intractable for most 2D commercial models currently available (e.g. RMA2, MIKE 21C, FESWMS).

Busnelli et al. (2001) argue that hydraulic jumps tend to rapidly disappear in alluvial beds under constant water discharge. The experiments reported by Bellal et al. (2003) show that amplitude of the hydraulic jump is drastically reduced by a movable sediment front. At least qualitatively, the model seems to show a rapid reduction in the intensity of the jump as bed evolves (Fig. 4). In the final profiles shown in Figure 5, the jump is barely noticeable. The agreement between measured and computed bed profiles is reasonable (Fig. 5), considering the large uncertainty in the sediment transport equation and the fact that both bank erosion and deposition have been completely ignored by assuming a fixed width. Brush & Wolman (1960) reported that channel became wider downstream of the knickpoint and narrower upstream; the maxima changes in width were about ±20% of the original width. Recently, Cantelli et al. (2004) and Wong et al. (2004) have demonstrated the importance of width changes in the morphological evolution of steep channels after dam removal; which can be considered as a knickpoint-like problem.

A very interesting and potential application of River2D-Morphology is the simulation of sediment transport after dam removal, which is a topic of high current interest (Cantelli et al. 2004, Wong et al. 2004, Cui & Wilcox, in press, Cui et al. a,b, in press). The hydraulic conditions after dam removal shown in Figure 1 (from Cui & Wilcox, in press) are practically identical to those depicted in Figure 3, with the over-steepened reach having a slope close the angle of repose of sediment. The computed bed profiles shown in Figure 4 also have some similarities with the initial bed profiles after dam removal computed by Cui & Wilcox (in press) for the Marmot Dam.

The main features of the model are:
- It uses a two-dimensional unstructured mesh based on the Finite Element method.
- It incorporates shock capturing techniques to simulate the sharp fronts of hydraulic jumps.
- It can dynamically update the bed elevation as scour and deposition progresses, in both subcritical and supercritical regimes.

Currently, the main limitations of the sediment transport model are:
- Suspended sediment is ignored.
- Sediment size is assumed uniform.
- Sediment waves (advection-dominated problems) cannot be properly simulated.

However, the present limitations listed above only reflect the early stage of development of this model, which is probably one of the first Finite Element depth-averaged models with capabilities for transcritical flow morphodynamics.

The potential of River2D-Morphology to simulate alluvial rivers in both lowland and mountain reaches looks promising.

REFERENCES


