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RIVER2D MORPHOLOGY, PART I: STRAIGHT ALLUVIAL CHANNELS

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ABSTRACT: We report the implementation and testing of a bedload sediment transport module that has been added to the 2D depth-averaged hydrodynamic model River2D. The purpose of this new River2D Morphology model is to simulate the morphodynamic changes of bed elevation in alluvial rivers. The model has been initially tested using three experimental cases of bed elevation changes in straight alluvial channels: (1) Bed aggradation due to sediment overloading; (2) bed degradation due to sediment feed shut-off (similar to degradation below a dam); and (3) knickpoint migration. The knickpoint migration experiment involved a short reach of 10% bed slope that caused supercritical flow and a hydraulic jump. The computed longitudinal profiles of bed and water surface elevations compare well with their measured counterparts, especially for the first two case studies. The model remained stable, even in the case of transcritical flow, challenging previous results of several researchers that have suggested that a decoupled model like this one (i.e. sediment equations are solved after flow equations) should become unstable when the Froude number approaches one or the boundary conditions change quickly. The model has proved to be reliable and stable for modeling straight alluvial channels. Applications to curved alluvial channels are shown in a companion paper (Part II).

1. INTRODUCTION

In a natural alluvial river an equilibrium condition may exist between the flow and sediment transport regimes such that the riverbed elevation remains practically constant. When this delicate equilibrium is altered, the riverbed responds by aggrading (sedimentation) or degrading (erosion) in an attempt to achieve a new equilibrium state. Riverbed aggradation may occur when the river is subject to a load of sediment higher than the flow is capable of transporting (Soni et al. 1980). Bed degradation is normally observed below of dams that trap sediment and produce a bedload deficit in the downstream reach. Sometimes, both aggradation and degradation are observed in a given reach, for example during a knickpoint migration (Brush and Wolman 1960). Such riverbed changes have several practical implications.

Bed aggradation could lead to higher water levels increasing the risk of flooding; also sediment depositions might affect navigability or riverine habitats. Bed degradation could undermine the foundations of constructed structures (e.g. bridges, dykes, pipelines) or drawdown surface water and groundwater levels affecting riparian habitats. Some current river restoration practices such as the removal of bank protection works (Schmautz 2004) or dams (Cui and Wilcox in press) can produce severe cases of bed aggradation and degradation with important economical and ecological implications. Those are some of the reasons why numerical modeling is becoming increasingly popular for simulating morphological changes in rivers.

We have developed and implemented a new bedload transport module into the existing two-dimensional (2D) hydrodynamic model River2D to simulate morphological changes in alluvial rivers. The application to straight alluvial channels is reported here, while application to curved channels is presented in a companion paper (Vasquez et al. 2005).

1.1. River2D hydrodynamic model

The River2D hydrodynamic model is a 2D depth averaged finite element model intended for use on natural streams and rivers and has special features for supercritical/subcritical flow transitions, ice covers, and variable wetted area. It has been developed by the University of Alberta and it is freely available at www.River2D.ca. River2D is based on the 2D depth averaged St. Venant Equations expressed in conservative form; which form a system of three equations representing the conservation of water mass and the two components of the momentum vector. The dependent variables solved for are the water depth and discharge intensities in the two respective coordinate directions (Steffler and Blackburn, 2002).

The Finite Element Method used in River2D's hydrodynamic model is based on the Streamline Upwind Petrov-Galerkin weighted residual formulation (Hicks and Steffler 1992). In this technique, upstream biased test functions are used to ensure solution stability under the full range of flow conditions, including subcritical, supercritical, and transcritical flow. A fully conservative discretization is implemented which ensures that no fluid mass is lost or gained over the modeled domain. This also allows implementation of boundary conditions as natural flow or forced conditions (Steffler and Blackburn, 2002).

1.2. Bedload sediment model

By considering only bedload transport and neglecting grain sorting, the 2D sediment continuity equation may be written as

[1] $$(1-\lambda)\frac{\partial z_b}{\partial t}+\frac{\partial q_{sx}}{\partial x}+\frac{\partial q_{sy}}{\partial y}=0$$

q_{sx} and q_{sy} are the components of the volumetric rate of bedload transport per unit length in the x and y directions (horizontal plane), λ is the porosity of the bed material (a default value of 0.4 is assumed), t is time and z_b the bed elevation. The bedload transport rate is a function of the flow hydraulics and sediment properties. A very simple approach is to assume that sediment transport depends only on velocity according to an empirical power law relationship of the form (Kassem and Chaudhry 1998)

[2a] $$q_{sx}=au(\sqrt{u^2+v^2})^{b-1}$$

[2b] $$q_{sy}=av(\sqrt{u^2+v^2})^{b-1}$$

a and b are empirical parameters, while u and v are the Cartesian components of depth-averaged velocity. Equation 2 usually provides good results for uniform sand. Other transport relations based on stream power or shear stress can also be easily coded in the software.

At every time step during the simulation, the hydrodynamic model computes water depth h and velocity components u and v. Those values are used in equation 2 to compute the sediment transport rates, which are applied in equation 1 to compute the changes in bed elevation Δz_b in a given time step Δt. A second order Runge-Kutta scheme is used to solve equation 1. The bed elevation is updated as $z_b^{new}=z_b^{old}+\Delta z_b$. Next, the hydrodynamic model is invoked again to recompute the flow field for the new bed topography. This procedure is repeated every time step until the final simulation time is achieved. This approach, in which sediment equations are solved after the flow equations, is usually referred as decoupled. In contrast, in a fully coupled approach equation 1 is solved simultaneously with the flow equations; z_b is treated as an additional unknown in four-by-four system (h, u, v, z_b). Also, in fully coupled models additional terms are added to both the flow and the sediment the continuity equations (Cao et al. 2002) to provide additional coupling between the flow variables (h, u, v) and the bed variable (z_b).

We chose the decoupled approach because it has several practical advantages. Starting from an advanced and mature hydrodynamic code (e.g. River2D), additional sub-models can be efficiently added in a modular and simple fashion. The sediment transport equation can take any desired level of complexity; which is very important for modeling graded sediment with multiple size fractions and armouring (gravel beds). However, decoupled models have been severely criticized as discussed below.

1.3. Coupled vs. decoupled models

There is an ongoing debate on which approach is the most suitable for morphological river modeling. Decoupled models have been criticized as been mathematically ill-posed and numerically unstable; incapable of handling rapidly changing boundary conditions or supercritical flow (Lyn 1987, Correia at al. 1992, Cao et al. 2002). However, Kassem and Chaudhry (1998) claimed that decoupled models are not unstable as usually referenced in the literature. Cui et al. (1996) reported "surprisingly good agreement" between coupled and decoupled models for the numerical simulation of aggradation and downstream fining; they suggest that concerns in the literature about the use of decoupled models may have been overstated. Except Kassem and Chaudhry (1998), most of the reported work was performed using simple 1D models.

This paper has two main purposes: to test the capabilities of the model for simulating three flume experiments in straight alluvial channels, while at the same assessing its numerical stability. Therefore, the experiments selected provide real observed data to objectively assess the model’s prediction capabilities, but also flow and sediment conditions that supposedly should make our decoupled model breakdown. The three experiments are: (1) bed aggradation due to sediment overloading (Soni et a. 1980), (2) bed degradation due to sediment feed shut-off (Suryanarayana 1969) and (3) knickpoint migration (Brush and Wolman 1960). Our results demonstrated that the computed bed profiles agreed reasonably well with measured data without any noticeable stability problem.

2. BED AGGRADATION TEST CASE

2.1. Soni et al. experiment

Soni et al. (1980) performed experiments of bed aggradation due to sediment overloading in a straight alluvial flume. The experimental flume was 30 m long and 0.20 m wide covered by uniform sand with a median diameter of 0.32 mm. From 24 measurements of sediment transport rate for flow velocities between 0.2 and 0.6 m/s, they found that values of a = 0.00145 and b = 5.0 in equation 2 resulted in good agreement with measured transport rates. Starting from equilibrium conditions the inflow of sediment q_{sIN} was increased up to several times the equilibrium value q_{se} in order to induce bed aggradation. The measured bed profiles were averaged because of the presence of ripples and dunes, especially in the non-aggrading reach. About 5% of the injected sediment was deposited upstream of the section of sediment addition.

For the numerical simulation we selected the experiment with the highest inflow of sediment $q_{sIN} = 4.0q_{se}$; with the initial conditions: bed slope $S_o = 0.036$; depth $h_o = 0.05$ m and a water discharge Q = 4 L/s. The flume was discretised using 336 triangular elements and 338 nodes. The average distance between nodes was set equal to the flume width $\Delta x = 0.20$ m. The roughness height was set to $k_s = 11$ mm and remained constant during all the simulation.

2.2. Lyn’s numerical experiment

Lyn (1987) performed a qualitative numerical experiment of bed aggradation in a fictitious reservoir under unsteady upstream boundary conditions for sediment inflow, as given by equation 3 (also applied by

Correia at al. 1992 in a similar analysis). According to equation 3, the sediment inflow varies periodically in time between 1 and 5 times the equilibrium value q_{se} during the course of the simulation time T.

We also performed a qualitative stability test applying equation 3 as upstream boundary condition for Soni et al. (1980) experimental flume, adopting T = 2400 s (40 min) as the total simulation time. All other parameters as described in section 2.1 remained unchanged.

[3] $$\frac{q_{sIN}(t)}{q_{se}} = 3 - 2\cos\left(2\pi\frac{t}{T}\right) \qquad 0 \le t \le T$$

2.3. Results

Figure 1 shows the results of the computed bed and water surface elevation (WSE) profiles at 40 minutes for Soni et al. (1980) experiment of bed aggradation. The time step used was Δt = 20 s. The Froude number computed in the upstream cross section became temporarily supercritical (Fr = 1.3) during the first minute, but returned to subcritical values afterwards.

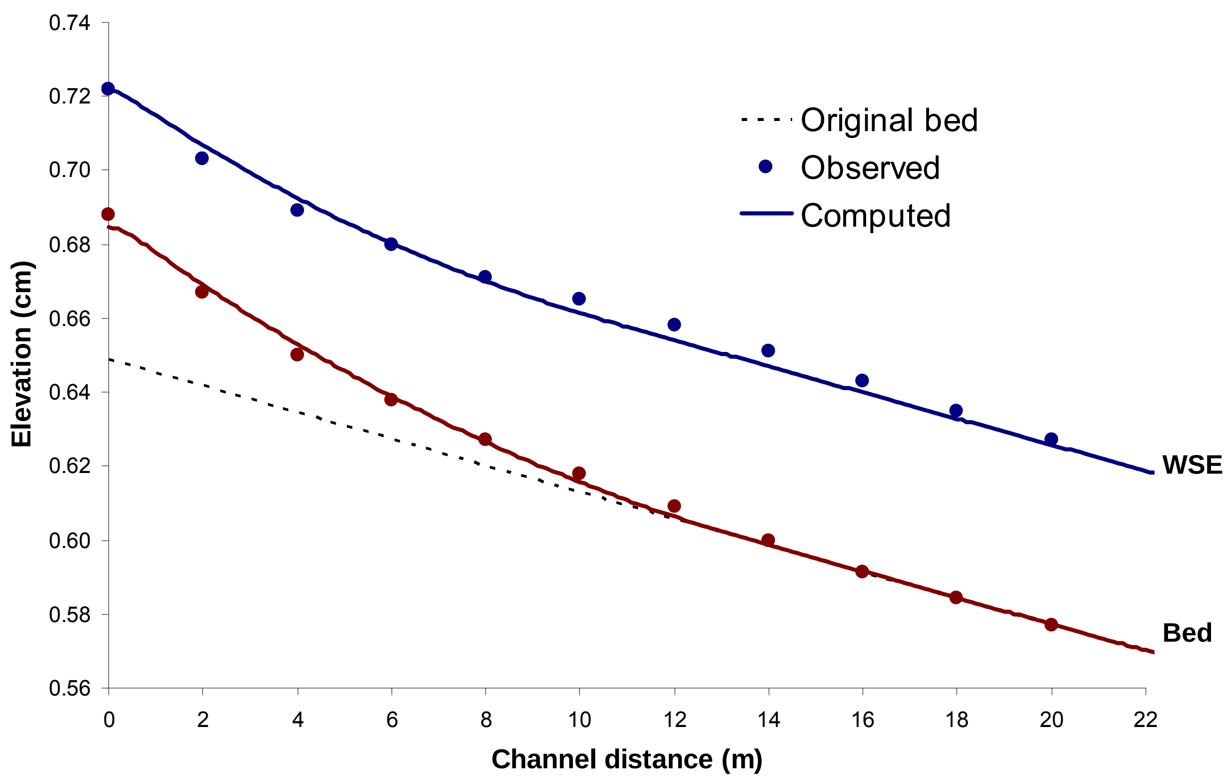

Figure 1. Observed and computed longitudinal profiles at 40 minutes for bed aggradation experiment due to an overload of sediment equal to four times the equilibrium value.

The computed profiles at 40 minutes using equation 3 as upstream boundary condition for sediment are shown in Fig. 2. The time step used was Δt = 20 s. Since this is a qualitative case, no measured data is available for comparison.

2.4. Discussion

There is good agreement between the computed and measured results for the aggradation case (Fig. 1); similar results were also obtained at intermediate times of 15 and 30 minutes (not shown). The difference between observed and computed values can be partially attributed to the varying bed roughness during the experiment due to the presence of bedforms (we assumed constant roughness), the uncertainty in the sediment transport equation, and the small fraction of injected sediment deposited upstream of the first cross section (ignored by the numerical model).

In a recent numerical simulation of this experiment, Cao at al. (2002) were forced to artificially reduce the friction factor to 80% relative to the steady-state case in order to obtain good agreement. We did not need such reduction. Cao et al. (2002) also reported that a decoupled model could not be applied to this case because it is not possible to define an upstream boundary condition for sediment transport when the flow becomes supercritical. Our results disagree with that conclusion; but agree with Kassem and Chaudhry (1998) who successfully applied a decoupled model for this problem. Similar results were also found by Cui et al. (1996) when simulating aggradation of heterogeneous gravel. They reported negligible differences between coupled and decoupled models to simulate bed aggradation despite having an upstream Froude number close to 1.0.

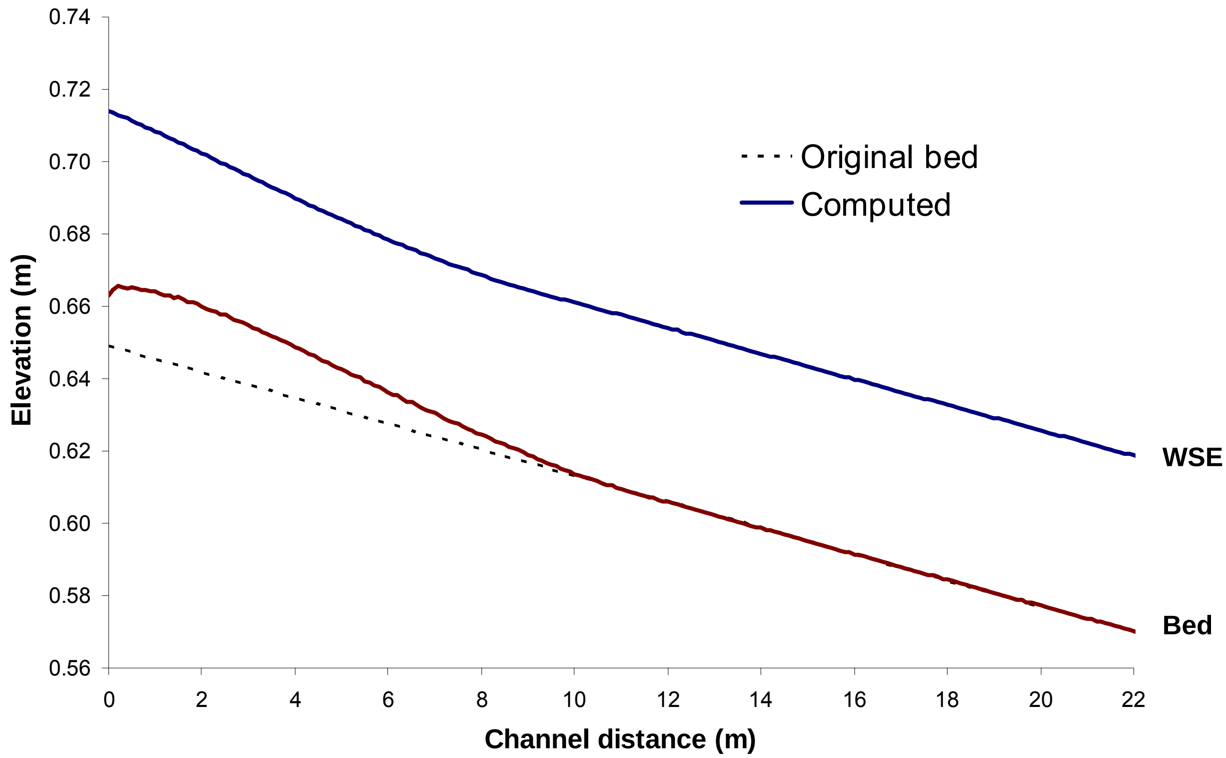

Figure 2. Computed aggradation profiles at 40 minutes with unsteady inflow of sediment as upstream boundary condition.

The hydrodynamic model uses a prescribed water surface elevation as downstream boundary condition. The same boundary condition is maintained when the morphological model is invoked to compute bed level changes. This is in clear disagreement with Cao et al. (2002) who stated that decoupled models require the flow depth to be specified, rather than the water surface elevation, as downstream boundary condition.

Park and Jain (1986) performed a numerical study of bed aggradation due to overloading using a 1D Finite Difference approach. They stated that stability requirements impose a limit on the time step (Park and Jain 1986, equation 28)

$$[4] \qquad \Delta t \leq \frac{3S_o(1-\lambda)}{2bq_{se}}(\Delta x)^2$$

Applying equation 4 with $\Delta x = 0.20$ m will lead to $\Delta t \leq 1.8$ s. However, our simulations remained stable with a time step about an order of magnitude higher: $\Delta t = 20$ s (Fig. 1 and 2). In contrast, Cao et al. (2002) for their simulations of with $\Delta x = 1.0$ m adopted $\Delta t = 0.6$ s, which is more than one order of magnitude smaller than the value predicted by equation 4.

From a theoretical and numerical analysis, Lyn (1987) concluded that decoupling leads to an ill-posed problem for which a general boundary condition cannot be satisfied. He stated that for problems in which rapid changes in both fluid and sediment discharge are imposed, coupling becomes necessary. However, our qualitative test case applying Lyn's equation 3 as upstream varying boundary condition remained stable; the computed profiles were smooth and free from any noticeable spurious oscillation as demonstrated in Fig. 2.

3. BED DEGRADATION TEST CASE

3.1. Suryanarayana experiment

Suryanarayana (1969) performed experiments of bed degradation due to the shut-off of upstream sediment supply; similar to the effect of a dam construction in a river. The flume was 18.29 m (60 ft) long and 0.61 m (2 ft) wide covered by uniform sand with a median diameter of 0.45 mm. The parameters used in equation 2 were $a = 1.786 \times 10^{-4}$ and $b = 3.878$ (Kassem and Chaudhry 1998). Starting from equilibrium conditions, the upstream sediment supply was shut off leading to bed degradation. The initial flow conditions of the selected test case were: $S_o = 0.007$, $h_o = 0.034$ m and $Q = 11.9$ L/s. The results were given for 10 hours of test time.

In the numerical model the flume was discretised using 70 triangular elements with 70 nodes. The average distance between nodes was set equal to the flume width $\Delta x = 0.61$ m. The roughness height was set equal to $k_s = 2.7$ mm. The upstream boundary condition was set to constant and equal to $q_{sIN} = 0$. Lu and Shen (1986) stated that since it takes a longer distance for clear water to pick up enough sediment from the channel bed to reach a high sediment transport capacity for a steeper slope condition, a reduction coefficient Cr = 0.4 must be introduced at the upstream end. An identical approach was also used by Kassem and Chaudhry (1998). In our numerical model we applied Cr to the sediment fluxes across the sides of the most upstream elements attached to the inflow cross section. Two simulations were performed for Cr = 1.0 (no reduction) and Cr = 0.4.

3.2. Results

Fig. 3 shows the computed bed and water surface profiles computed at 10 hours after the beginning of the bed degradation experiment. The time step used was $\Delta t = 30$ s. The Froude number remained subcritical during the entire simulation.

3.3. Discussion

There is a reasonable agreement between the measured and computed bed and water surface profiles as shown in Fig. 3. The agreement was less accurate in the upstream reach where an adverse bed slope is present. Bedload sediment moving along this adverse slope is subject to a component of weight that acts against the direction of the flow, reducing sediment mobility. Such sloping effect is ignored by the simple equation 2 which only depends on velocity. This effect is somehow taken into account by the correction coefficient Cr applied to the upstream section (Lu and Shen 1986, Kassem and Chaudhry 1998). If the sediment flux is not reduced upstream (Cr = 1.0), the computed profiles almost fit perfectly the observed profiles in the last two thirds of the flume (Channel distance > 6 m in Fig. 3); however, the WSE is under-predicted in the upstream sections. When we applied Cr = 0.4 (Kassem and Chaudhry 1998, Lu and Shen 1986) a better fit was obtained in the upstream reach, but to the detriment of the computed values in the downstream reach.

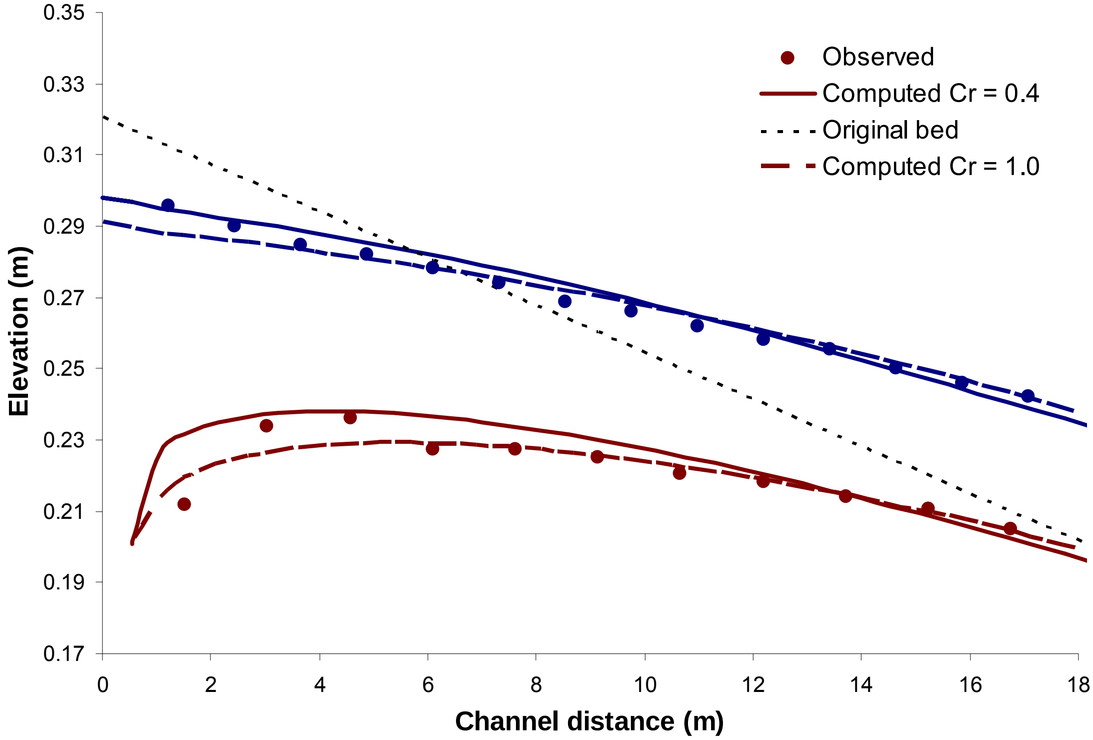

Figure 3. Observed and computed longitudinal profiles at 10 hours for bed degradation experiment due to sediment supply shut-off for two correction coefficient values Cr = 1.0 and Cr = 0.4.

4. KNICKPOINT MIGRATION TEST CASE

4.1. Brush and Wolman experiment

Knickpoints are points along longitudinal profiles of streams where the slope increases abruptly. When they are present on alluvial channels, in an attempt to reduce the bed slope the flow erodes the steep bed slope reach causing the knickpoint to migrate upstream. Brush and Wolman (1960) carried out experiments in a flume 15.85 m (52 ft) long and 1.22 m (4 ft) wide, where a trapezoidal channel 0.21 m wide was carved before starting each experimental run. The longitudinal bed slope changed at a knickpoint from 0.125% to10% and back to 0.125% again (Fig. 4). The short oversteepened reach had a length of 0.30 m (1 ft) and was located 10.8 m from the flume entrance. Five test runs were performed; we selected Run 1 for the numerical simulations. For Run 1 the channel was moulded in noncohesive sand 0.67 mm in diameter with initial conditions h_o = 0.0137 m and Q = 0.59 L/s. During Run 1 the width of the channel increased downstream of the knickpoint by bank erosion to about 0.25 m; while it got deeper and narrower upstream.

In the numerical model the channel was assumed as rectangular with a fixed width of 0.21 m. Unlike the two previous test cases, we used a more irregular mesh; the average size of the elements was about 0.20 m away from the steep reach and about 0.05 m around the knickpoint. The mesh was refined close to the knickpoint to capture the strong flow gradients and the hydraulic jump. The mesh had 869 triangular elements and 563 nodes. The roughness height was set to k_s = 0.3 mm.

Given that most sediment transport equations are derived for subcritical uniform flow conditions there is considerable uncertainty in this case, where supercritical flow occurs. Bhallamudi and Chaudhry (1991) adopted $a = (S_e)^{-1.7}$ and b = 4.2 in equation 2, being S_e the energy slope. We applied the same values assuming the energy slope could be approximated by the Chezy friction equation.

4.2. Results

Fig. 4 shows the initial water surface profile computed before the beginning of the morphological simulation. The Froude number varied between 0.5 and 3.5 with a hydraulic jump downstream of the toe of the oversteepened reach. The water depth varied between 0.004 and 0.014 m.

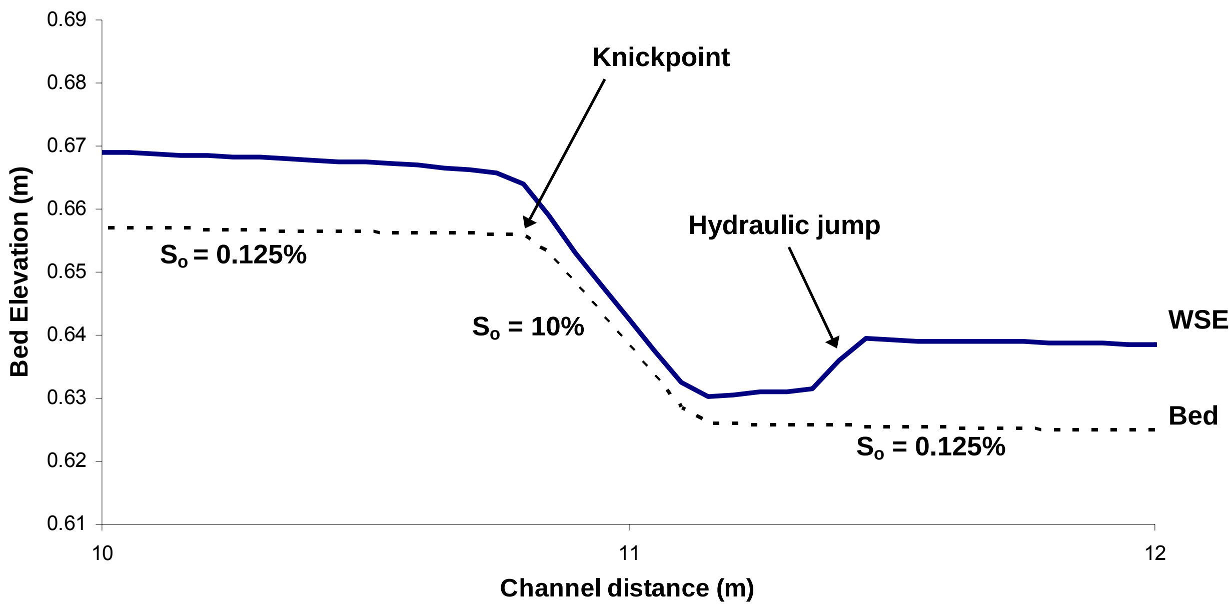

Figure 4. Initial conditions for knickpoint migration experiment (t = 0). Water surface elevation was computed by the numerical model.

The computed bed and water surface profiles after 2 h 40 min are shown in Fig. 5 together with the bed level measurements. The time step was kept small Δt = 0.01s to avoid large bed changes during the initial period of the simulation, when most intensive sediment transport occurred. Both the computed bed and WSE profiles were rather smooth, without any noticeable numerical oscillation. At this time, the maximum value of the Froude number was only slightly larger than 1.0 because the steep bed slope was reduced to about 0.44%. The new location of the knickpoint and upstream bed profile were well reproduced; but not so in the downstream reach. In general, the computed bed profile seems to follow the average trend of the measured bed levels.

4.3. Discussion

This knickpoint migration experiment is a fairly challenging problem because of the steep 10% bed slope, which is at the limit of what River2D can model correctly (Steffler and Blackburn 2002), plus the presence of a hydraulic jump. Although transcritical flow capabilities are standard in 1D models (e.g. HEC-RAS, BRI-STARS); they are still uncommon in most 2D and 3D models (e.g. MIKE 21, RMA2, FESWMS, SSIIM). The shock-capturing techniques incorporated in River2D, added to the flexibility of the Finite

Element method to increase mesh resolution around areas with strong gradients, make possible to simulate the sharp hydraulic jump front (Fig. 4), without any numerical oscillation. This test case would be intractable for most 2D commercial models currently available.

The agreement between the measured and computed bed profiles is at least reasonable, given the great uncertainty in the sediment transport equation and the fact that bank erosion/deposition has been completely ignored (i.e. fixed width simulation). In fact, the most important result of this test case is that the River2D-Morphology remained stable despite the presence of supercritical flow and a hydraulic jump that caused intense and rapid bed changes, especially during the early stages of the simulation.

A very interesting and promising application of River2D-Morphology to a knickpoint-like problem is the simulation of sediment transport after dam removal, which is a topic of high current interest in North America (Cui and Wilcox in press). The hydraulic conditions after dam removal are practically identical to those depicted in Fig. 4, with the oversteepened reach having a slope close the angle of repose, usually comprising graded sediment (sand and gravel). To our best knowledge, 2D models with capabilities for modeling the transport of graded sediment under such steep bed slopes are not presently available. Since we are using a decoupled approach, it will be relatively simple to include complex sediment transport predictors for multiple size fractions in future versions of River2D-Morphology.

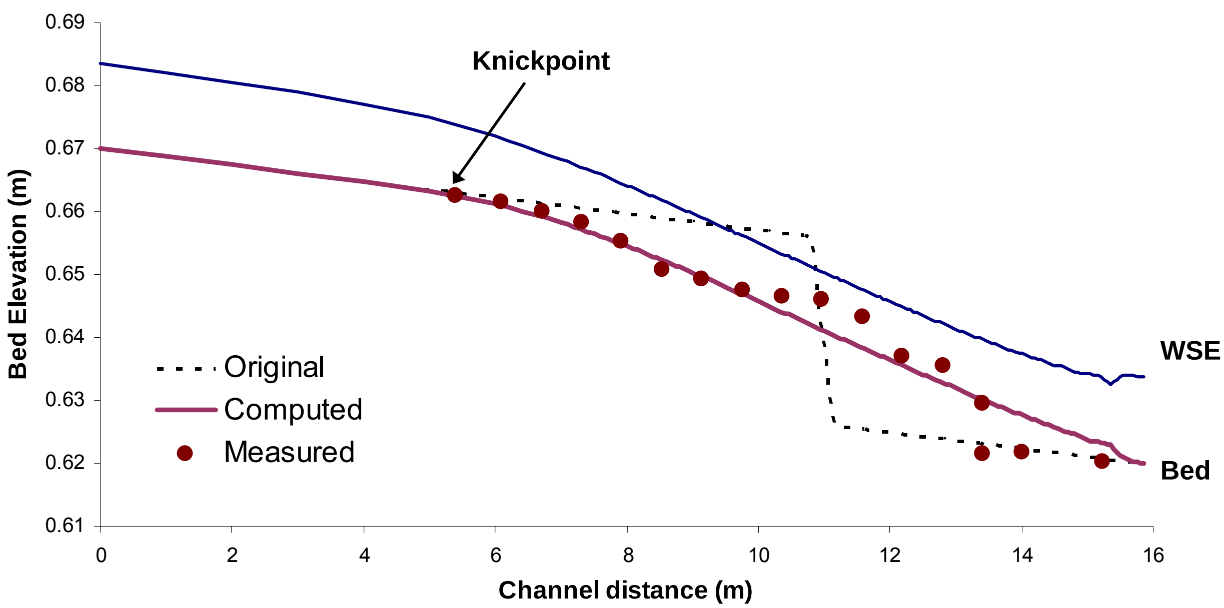

Figure 5. Observed and computed longitudinal profiles at t = 2 hr 40 min for knickpoint migration experiment.

5. SUMMARY AND CONCLUSIONS

We report the first application of the recently developed River2D-MORphology model. River2D-MOR was developed by adding a solver for the bedload sediment continuity equation to the existing River2D hydrodynamic model; making it possible to update the channel bed elevation as sediment is eroded or deposited by the flow. The sediment transport equations are solved after the flow equations, in a fashion usually referred as decoupled.

River2D-MOR has demonstrated its capabilities to successfully simulate bed level changes in alluvial straight channels, even in cases with rapidly varying boundary conditions, supercritical flow and hydraulic jumps. The model remained stable and provided satisfactory agreement for three experimental test cases: (1) bed aggradation, (2) bed degradation and (3) knickpoint migration.

Our results challenged conclusions of several previous researchers who have heavily criticized decoupled models stating that they are unstable, not capable of simulating rapidly changing boundary conditions or supercritical flow (Lyn 1987, Correia et al. 1992, Cao et al. 2002). None of our results showed any of those problems; even when relative coarse meshes and large time steps were used. In general, we agree with the conclusion of Cui et al. (1996) that concerns in the literature about the use of decoupled models may have been overstated. In agreement with Kassem and Chaudhry (1998), it appears that the stability problems reported by previous researchers might be due to the numerical approach they employed.

The model is currently limited to bedload dominated flows (suspended sediment not important or negligible) and uniform sediment. Research to include additional sub-models for graded sediment (sand and gravel) is currently underway. Applications of River2D-MOR to curved alluvial channels subject to secondary flow conditions are presented in a companion paper (Vasquez et al. 2005).

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